Note on Henrard swaption evaluation

Table of contents

[I. Henrard decomposition for swaptions 2](#_Toc388545262)

[A. Payoff and notations 2](#_Toc388545263)

[B. Derivation of the formula 2](#_Toc388545264)

[C. Monotonicity of the integrand 4](#_Toc388545265)

[D. Swaption call/put prices 6](#_Toc388545266)

[II. Application to multi-curve 6](#_Toc388545267)

[A. Multi-curve approximation 6](#_Toc388545268)

[B. Application to swaption pricing 7](#_Toc388545269)

[III. Application to digital options 9](#_Toc388545270)

[IV. Numerical integration 9](#_Toc388545271)

[References 10](#_Toc388545272)

This document details the computation leading to the swaption pricing formula derived by **(Henrard, 2009)**, and describes how it is used in the context of multi-curve swaption and digital swaption computations.

# Henrard decomposition for swaptions

## Payoff and notations

* Swaption value a t: or or (generic for call or put)
* Option maturity:
* Annuity:
* Fixed leg payment dates: ,
* Bond price: with
* Forward Swap Rate:
* Money market account

In a mono-curve framework:

## Derivation of the formula

At maturity, for a put:

with

Similarly, for a call:

with

We will denote by the swaption value with no reference to the call or put feature:

Change of numeraire:

Under the -forward numeraire, is a martingale with dynamic:

is independent of  only one factor enters into the expression of the swaption

Where and

## Monotonicity of the integrand

Let be such that for and for

We use:

To simplify the analysis, we use for

is an increasing function of , to simplify we use:

* for
* for

Which leads to:

is the subtraction of:

* : a decreasing function:
* : an increasing (or constant if function:

Thus:

* is a decreasing function of

Which implies that

A similar analysis for the case where is such that for and for leads to:

## Swaption call/put prices

Let be such that

Recall that for a call swaption, we have:

Ie: for and for

|  |  |  |
| --- | --- | --- |
|  |  |  |

For a put swaption, we have:

Ie: for and for

|  |  |  |
| --- | --- | --- |
|  |  |  |

# Application to multi-curve

## Multi-curve approximation

In a multi-curve framework:

is the libor computed on the specific evaluation curve

is the libor computed on the discount curve

is the zero coupon bond computed on the discount curve

The approximation used (not an approximation when ) is to freeze the libor adjustments:

The HW model is calibrated on a reference rate curve which may be different from the discount and evaluation rate curves. Because we assume that the spreads between the evaluation/discount rate curves and the reference curve are constant, we have:

Where is the zero coupon bond price deduced from HW model.

We can thus drop the superscript without loss of generality, by modifying accordingly the coefficients

## Application to swaption pricing

* **Call swaption**

Where:

* **Put swaption**

For a put swaption, we have:

Provided for and for , we can use the swaption call formula (1).

Provided for and for , we can use the swaption put formula (2).

Otherwise, we will integrate numerically the formula

# Application to digital options

The price of digital options can be derived similarly:

When follow the same conditions as the standard call and put swaptions,

such that

The digital call and put options formulae are derived straightforwardly

* **Digital call option**

|  |  |  |
| --- | --- | --- |
|  |  |  |

* **Digital put option**

|  |  |  |
| --- | --- | --- |
|  |  |  |

# Numerical integration

When we are not in a case where change of sign only once, we need to compute numerically the integral:

We use a Gauss-Legendre integration with 100 points and with bounds equal to and standard deviations:

In order to control the error between the theoretical and numerical values, we add a correction term:

* We degenerate the initial swaption such that there is only one change of sign (ie: all between the 1st change of sign and the last change of sign are set to 0.
* The correction term is equal to the difference between the theoretical and numerical values of this degenerate swaption.

**Example:**

# References

Henrard, M. (2009). Efficient swaptions price in Hull-White one factor model.